Parameter Estimation and Target Detection of Phased-MIMO Radar Using Capon Estimator

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Abstract

Phased-Multiple Input Multiple Output (PMIMO) radar is multi-antenna radar that combines the main advantages of the phased array (PA) and the MIMO radars. The advantage of the PA radar is that it has a high directional coherent gain making it suitable for detecting distant and small radar cross-section (RCS) targets. Meanwhile, the main advantage of the MIMO radar is its high waveform diversity gain which makes it suitable for detecting multiple targets. The combination of these advantages is manifested by the use of overlapping subarrays in the transmit (Tx) array to improve the performance of parameters such as angle resolution and detection accuracy at amplitude and phase proportional to the maximum number of detectable targets. This paper derives a parameter estimation formula with Capon's adaptive estimator and evaluates it for the performance of these parameters. Likewise, derivation for expressions of detection performance such as the probability of false alarm and the probability of detection is also given. The effectiveness and validation of its performance are compared to conventional estimator for other types of radars in terms of the effect of the number of target angles, the RCS of targets, and variations in the number of subarrays at Tx of this radar. Meanwhile, the detection performance is evaluated based on the effect of Signal to Noise Ratio (SNR) and the number of subarrays at Tx. The evaluation results of the estimator show that it is superior to the conventional estimator for estimating the parameters of this radar as well as the detection performance. Having no sidelobe makes this estimator strong against the influence of interference and jamming so that it is suitable and attractive for the design of radar systems. Root mean square error (RMSE) on magnitude detection from LS and Capon estimators were 0.033 and 0.062, respectively. Meanwhile, the detection performance for this radar has the probability of false alarm above 10^{-4} and the probability of detection of more than 99%.

Keywords: Capon estimator, MIMO radar, phased-array antenna, subarrays, target detection.

I. INTRODUCTION

The development of radar system technology with multiple antennas has been going on since the 1930s for various applications. Like a vehicular radar, a radar with high accuracy and resolution for the detection of multiple targets is absolutely necessary [1], [2]. As a radar for detecting vital human organs, a radar that detects multiple targets with slow motion is needed [3] as well as for other applications.

Currently, in addition to the phased array (PA) and the Multiple Input Multiple Output (MIMO) radar, there is also a radar that combines the main advantages of the two radars called the Phased-MIMO radar (PMIMO). The radar research was pioneered by [4]. This radar utilizes the MIMO radar principle where array elements are in the form of overlapping subarrays on the Transmit array (Tx) which has a performance like the PA radar. One subarray with the other is orthogonal so they utilize the advantages of waveform diversity gain, such as the MIMO radar, to detect multiple targets while each subarray has high directional coherent gain advantages, such as the PA radar, to detect targets that have weak or

small Radar Cross Section (RCS). So, based on the combination of these advantages, this radar has parameter detection between two extreme antenna configurations, i.e. the PA and MIMO radar [2]. Besides, a combination of signal processing and adaptive array techniques, this radar will form a large virtual aperture array, increase the resolution of spatial spectrum estimation, and significantly improve parameter identifiability.

Detection parameters especially such as the maximum number of detection targets on the PMIMO radar have been reported by [5]. The study uses a conventional estimator method called the least squares (LS) estimator. In this estimation, it turns out that there are weaknesses, i.e. having high sidelobes and low resolution [6]. The estimator also has a weakness in the accuracy of amplitude detection, which is proportional to RCS, and angular resolution. Detection performance is strongly influenced by the angular resolution supported by low sidelobe levels such as the use of certain waveforms [7]. Under conditions of detection with strong interference and jamming, the method does not function properly. To overcome this problem, adaptive estimators are used. There are many adaptive estimators such as Capon, amplitude, and phase estimation (APES), etc [8].

One of the adaptive estimators used to detect the location of targets and the complex amplitude of reflected

signals with a better angular resolution is the Capon estimator. This estimator is presented in this paper. Through numerical calculations, it is shown that this estimator has a better estimate than the previous estimator. Unlike traditional estimators, this estimator can estimate amplitude more accurately as well as its angular resolution. Furthermore, the performance of target detection with this estimator on this radar is analyzed. [5] has estimated the parameters of this radar with the LS method then compared its performance against other radars such as the PA and the MIMO radars. However, the study has not yet discussed in detail the estimation of the effect of target angle resolution, the variation of the target RCS, and variations in the number of subarrays at Tx of this radar where these are described in this paper. Furthermore, a comparison of the effectiveness of performance between this estimator and traditional estimator regarding parameter estimation is presented in this paper. The results of this study found that the target estimation is high resolution and has the ability to suppress interference and jamming better than conventional techniques.

This paper is an extension of the detection performance formulation on the MIMO radar reported by [11] where the spacing between antenna elements on the radar is widely separated whereas the study is co-located. The target detection reported by [11] uses the likelihood ratio test (LRT) approach with optimal detection based on Neyman-Pearson criteria. After the detection performance expressions of this radar are obtained, namely the probability of detection and the probability of false alarms, evaluation and validation are carried out on them at the same time comparing their performance against other radars by considering aspects such as Signal to Noise Ratio (SNR) and the number of subarrays in Tx. Thus, the main contributions of this paper that have not been previously reported, including by [5], [8], and [11] are summarized as:

1) The formulation and evaluation of parameter estimation on the PMIMO radar uses the Capon estimator approach because in [5] only the LS estimator is applied to all radars while in [8] the type of radar is the MIMO radar.

2) The parameter estimation using the Capon method considers the effect of target angle resolution, the variation of the target RCS, and variation in the number of subarrays at Tx of this radar including the influence of the jammer.

3) Expression and evaluation of the probability of detection and the probability of false alarm with the LRT approach for this radar by considering the effect of SNR and the number of subarrays at Tx.

This paper is organized as follows. Section II reviews the system model and signals from the PMIMO radar. Section III presents a description of the Capon estimator for parameter estimation on this radar and its detection performance with the LRT approach. Some examples of numerical simulations, evaluations, and analyzes are given in Section IV. Finally, this paper is concluded in Section V.

II. RADAR PHASED-MIMO

A. System Model

The PMIMO radar is the MIMO radar whose elements are overlapping subarrays that function as the PA. Assuming a radar system with collocated antennas has U elements in the transmit array (Tx) and V elements in the receive array (Rx). The distance between the antenna elements on Tx and Rx is d_U and d_V, respectively. In the Tx array, K subarrays are formed. The number of antenna elements in each subarray is $U_K = U - K + 1$. The transmitted signal is a narrow band and the propagation is non-dispersive.

On PMIMO radars, the K subarrays, as elements in Tx array, forms orthogonal waveforms which simultaneously compromise the main advantages between the PA radar, i.e. the directional coherent gain, and the MIMO radar, i.e. the waveform diversity gain, while V elements in Rx operate as independent receivers so that it can detect multiple targets. According to [2], this radar has a virtual array size, KV where K is $1 \leq K \leq U$. For example, in the case of Figure 1, this radar configuration has the same number of elements per subarray with $K = 3$ at Tx array, so that it produces 3 orthogonal waveforms that are transmitted by 3 subarrays at Tx and received by V elements at Rx.

B. Signal Model

The PMIMO radar diagram block is presented in Figure 1. All subarrays in Tx radiate orthogonal waveforms simultaneously towards multiple targets i.e. targets: $p = 1, 2, ..., P$. The k-subarray in the Tx array transmits the signal $\varphi_k(t)$ which is independent of the waveforms of the other subarray. The U elements in the Tx array form k overlapping subarrays which transmit the baseband signal vector with $k = 1, 2, ..., U$, i.e. (1)

$$\mathbf{x}_k(t) = \sqrt{U/K}\,\varphi_k(t)\mathbf{w}_k^*$$

(1)

where $(\cdot)^*$ is a complex conjugate operator, U/K is the power normalization coefficient which ensures that the energy transmitted by this radar in one pulse is U, $\mathbf{w}_k$ is the vector U element of the complex weight of the normal unit for the k-th subarray.

The signal reflected by the target located in θ, i.e. on the far field, with the reflection coefficient $\alpha(\theta)$ expressed by (2)

$$\mathbf{r}(t,\theta) = \sqrt{U/K}\,\alpha(\theta)\sum_{k=1}^{K}\mathbf{w}_k^H\mathbf{a}_k(\theta)e^{-j2\pi f\tau_k(\theta)}\,\varphi_k(t)$$

(2)

where $(\cdot)^H$ is the Hermitian transpose operator, $\mathbf{a}_k(\theta)$ is the steering vector Tx with U elements in the k-subarray, f is the carrier signal frequency, $\tau_k(\theta)$ is the relative delay time of the first element in the k-subarray with respect to the element first of the first subarray with $\tau_k(\theta) = kd_U \sin(\theta)/c$, and c is the speed of light in a vacuum ($\approx 3{\times}10^8$ m/s).
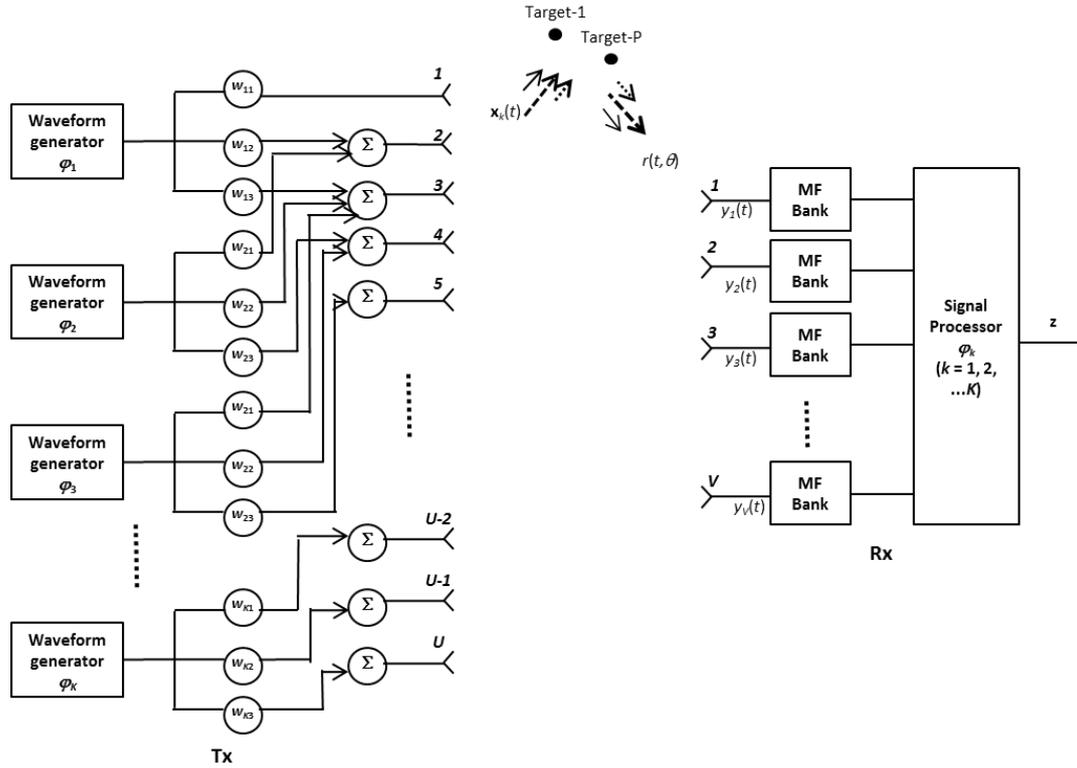
Figure 1. Illustration of The PMIMO Radar System for 3 Antenna Elements per Subarray with MF Denote Match Filter.

If it is defined that the coherent vector Tx and the diversity vector Tx of the K-element are, respectively, as (3) and (4)

$$\mathbf{c}(\theta) = [\mathbf{w}_1^H \mathbf{a}_1(\theta) \quad \mathbf{w}_2^H \mathbf{a}_2(\theta) \quad \cdots \quad \mathbf{w}_K^H \mathbf{a}_K(\theta)]^T \tag{3}$$

$$\mathbf{d}(\theta) = [e^{-j2\pi f \tau_1(\theta)} \quad e^{-j2\pi f \tau_2(\theta)} \quad \dots \quad e^{-j2\pi f \tau_K(\theta)}]^T \tag{4}$$

then (2) can be simplified into (5)

$$\mathbf{r}(t,\theta) = \sqrt{U/K} \alpha(\theta) \big(\mathbf{c}(\theta) {}^\circ \mathbf{d}(\theta)\big)^T \boldsymbol{\psi}(t) \tag{5}$$

where $(\bullet)^T$ is the transpose operator, $\boldsymbol{\psi}(t) = [\varphi_1(t) \, \varphi_2(t) \dots \varphi_K(t)]^T$ and $\circ$ are Hadamard's multiplication operators.

Assuming there is a P target in the direction of $\{\theta_p\}$, then the complex vector of the signal received by V subarray in Rx as (6)

$$\mathbf{y}(t) = \sqrt{U/K} \sum_{p=1}^P \alpha_p(\theta_p) \mathbf{b}(\theta_p) [\mathbf{c}(\theta_p) {}^\circ \mathbf{d}(\theta_p)]^T \boldsymbol{\psi}(t) + \mathbf{n}(t) \tag{6}$$

where $\mathbf{b}(\theta_p)$ is the Rx steering vector with V elements in the k-subarray and $\mathbf{n}(t)$ is an N-element vector of white Gaussian noise with zero means, interference, and jamming.

Furthermore, by using matched filter banks (MF) to separate the waveform $\varphi_k(t)$, the vector data $KV \times 1$ is generated, i.e. (7)

$$\mathbf{z} = [\mathbf{y}_1^T \quad \mathbf{y}_2^T \quad \cdots \quad \mathbf{y}_K^T]^T$$
$$= \sqrt{U/K} \sum_{p=1}^P \alpha_p(\theta_p) \mathbf{u}(\theta_p) + \mathbf{n} \tag{7}$$

with KV-elements of the Tx-Rx steering vector i.e. (8)

$$\mathbf{u}(\theta) = \big(\mathbf{c}(\theta) {}^\circ \mathbf{d}(\theta)\big) \otimes \mathbf{b}(\theta) \tag{8}$$

where $\otimes$ denotes the Kronecker multiplication operator.

III. PARAMETER ESTIMATION AND DETECTION PERFORMANCE FOR THE PMIMO RADAR

A. Parameter Estimation of the Capon Method

Based on the derivation of the parameter estimation by the Capon method, i.e. the maximum number of detectable targets, on the MIMO radar by [9], the formulation of the maximum number of detectable targets on the PMIMO radar is carried out with similar derivation stages. The following are the stages of the Capon method used to estimate the number of detectable targets on the PMIMO radar in (6), i.e.: (a) determination of the Capon beamforming and (b) target estimation $\hat{\alpha}(\theta)$ such as the LS method [9].

Beamformer from Capon method is formulated as (9)

$$\min_{\mathbf{w}} \mathbf{w}^H \widehat{\mathbf{R}}_{\mathbf{yy}} \mathbf{w} \quad \text{subject to} \quad \mathbf{w}^H \mathbf{b}(\theta) = 1 \qquad (9)$$

where $\mathbf{w} \in C^{V \times 1}$ is a weight vector to increase noise, interference, and jamming suppression so that the desired signal is not distorted, $\widehat{\mathbf{R}}_{\mathbf{yy}}$ is the covariance matrix of the data signal received on this radar expressed by (10)

$$\widehat{\mathbf{R}}_{\mathbf{yy}} = (1/Q) \sum_{q=1}^{Q} \mathbf{y}(q) \mathbf{y}^H(q) \qquad (10)$$

where $q = 1, 2, ..., Q$ is the index of the sample data.

The beamformer weight vector in the Capon method of the PMIMO radar is stated by (11)

$$\widehat{\mathbf{w}} = \frac{\widehat{\mathbf{R}}_{\mathbf{yy}}^{-1} \mathbf{b}(\theta)}{\mathbf{b}^H(\theta) \widehat{\mathbf{R}}_{\mathbf{yy}}^{-1} \mathbf{b}(\theta)} \qquad (11)$$

so, the beamformer output in the Capon method for the received data signal vector is (12).

$$\widehat{\mathbf{w}}^H \mathbf{y}(t) = \frac{\mathbf{b}^H(\theta) \widehat{\mathbf{R}}_{\mathbf{yy}}^{-1} \mathbf{y}(t)}{\mathbf{b}^H(\theta) \widehat{\mathbf{R}}_{\mathbf{yy}}^{-1} \mathbf{b}(\theta)} \qquad (12)$$

Then apply the LS method in (12) so that an estimation of the detection target for the Capon method on the PMIMO radar is obtained as (13)-(15)

$$\hat{\alpha}(\theta)$$
$$= \frac{\sqrt{U/K} \sum_{q=1}^{Q} \mathbf{b}^H(\theta) \widehat{\mathbf{R}}_{\mathbf{yy}}^{-1} \widehat{\mathbf{R}}_{\mathbf{y\psi}}^{-1} [\mathbf{c}(\theta)^\circ \mathbf{d}(\theta)]^*}{\mathbf{b}^H(\theta) \widehat{\mathbf{R}}_{\mathbf{yy}}^{-1} \mathbf{b}(\theta) [\mathbf{c}(\theta)^\circ \mathbf{d}(\theta)]^T \widehat{\mathbf{R}}_{\mathbf{\psi\psi}} [\mathbf{c}(\theta)^\circ \mathbf{d}(\theta)]} \qquad (13)$$

$$\widehat{\mathbf{R}}_{\mathbf{y\psi}} = (1/Q) \sum_{q=1}^{Q} \mathbf{y}(q) \mathbf{\psi}^H(q) \qquad (14)$$

$$\widehat{\mathbf{R}}_{\mathbf{\psi\psi}} = (1/Q) \sum_{q=1}^{Q} \mathbf{\psi}(q) \mathbf{\psi}^H(q) \qquad (15)$$

where $\hat{\alpha}(\theta)$ is the Capon estimator for the radar reflection coefficient or magnitude of complex amplitude (MCA) of the target in the direction of θ. The number of resolvable targets from these estimates can be identified from the peaks of the spatial spectrum.

The expression of parameter estimation using the Capon method on PMIMO radar which involves the use of overlapped subarrays is one of the main contributions to this paper. As a validation of the formulation. For the MIMO radar configuration with Tx array i.e. $K = U$ and Rx array i.e. V so that $\mathbf{a}(\theta) = \mathbf{1}_{K \times 1}$, $\mathbf{d}(\theta) = \mathbf{a}(\theta)$ and $\mathbf{b}(\theta) = \mathbf{1}_{V \times 1}$ then at (13) can simplified to be (16).

$$\hat{\alpha}(\theta) = \frac{\sum_{q=1}^{Q} \mathbf{b}^H(\theta) \widehat{\mathbf{R}}_{\mathbf{yy}}^{-1} \widehat{\mathbf{R}}_{\mathbf{y\psi}}^{-1} \mathbf{a}^*(\theta)}{\mathbf{b}^H(\theta) \widehat{\mathbf{R}}_{\mathbf{yy}}^{-1} \mathbf{b}(\theta) \mathbf{a}(\theta)^T \widehat{\mathbf{R}}_{\mathbf{\psi\psi}} \mathbf{a}^*(\theta)} \qquad (16)$$

Looks at (16) are in line with the results obtained by [8, (10)] and [9, Ch. 1, (1.36)] written in different ways.

The same thing was applied to (13) for the PA radar with Tx array i.e. $K = 1$ and Rx array i.e. V consequently $\psi(t) = \varphi_1(t)$, $\mathbf{a}(\theta_1) = \mathbf{a}(\theta)$, $\mathbf{d}(\theta) = 1$, and $\mathbf{b}(\theta) = \mathbf{1}_{V \times 1}$ so that it is obtained as (17).

$$\hat{\alpha}(\theta) = \frac{\sqrt{U} \sum_{q=1}^{Q} \mathbf{b}^H(\theta) \widehat{\mathbf{R}}_{\mathbf{yy}}^{-1} \hat{\mathbf{r}}_{\mathbf{y\psi}}^{-1} c^*(\theta)}{\mathbf{b}^H(\theta) \widehat{\mathbf{R}}_{\mathbf{yy}}^{-1} \mathbf{b}(\theta) c(\theta) c^*(\theta)} \qquad (17)$$

The comparison between (13) and (16) is the PMIMO radar with K subarrays on the Tx array, where $1 \leq K \leq U$, provides high flexibility to control the sidelobe level on the signal transmitted to and received from the target.

B. Detection Performance

The detection problem for the PMIMO radar is an extension of the study by [10]-[11] on the PA and the MIMO radars where the MIMO coherent is not the MIMO statistical so that it has an exponential distribution or according to [11] has a Chi-squared distribution with a DoF of $2 \sim \chi_2^2$. As the derivation of the expressions for target detection performance on the PA and the MIMO radars by [11], if the signal vector received by the PMIMO radar in (7) is passed MF and $\mathbf{u}(\theta)$ as in (8) then the detection problem is formulated as (18)

$$\begin{aligned} H_0: & \quad \mathbf{z} = \mathbf{n} & \text{Target doesn't exist} \\ H_1: & \quad \mathbf{z} = \sqrt{U/K} \alpha_s \mathbf{u}(\theta) + \mathbf{n} & \text{Target exist} \end{aligned} \qquad (18)$$

where H_0 and H_1 are the hypotheses that there is only noise without an echo signal at $\mathbf{z}$ and the hypothesis that there is an echo signal at $\mathbf{z}$. For $\mathbf{n} \sim \mathcal{CN}(0, \alpha_n^2 I_{KV})$ and $\mathbf{u}(\theta) \sim \mathcal{CN}(0, \alpha_s^2 I_{KV})$.

As in the MIMO and the PA radars [11], the optimum solution for hypothesis testing with the Neyman-Pearson criteria, i.e. the likelihood ratio test (LRT), requires knowledge of the probability distribution at $\mathbf{u}(\theta)$, i.e. (19)

$$\frac{\max_{\mathbf{u}(\theta)} P(\mathbf{z}|H_1, \alpha_n^2, \mathbf{u}(\theta))}{P(\mathbf{z}|H_0, \alpha_n^2)} \overset{>_{H_1}}{\underset{<_{H_0}}{}} \delta \qquad (19)$$

where δ is the threshold to determine the target detection set according to the desired false alarm rate [11].

For the probability density from $\mathbf{z}$ to H_1, it can be expressed as (20)-(21).

$$P(\mathbf{z}|H_1, \alpha_n^2, \mathbf{u}(\theta)) = \frac{1}{(\pi \alpha_n^2)^{KV}} exp(-\mathbf{z}_n^H \mathbf{z}_n \alpha_n^{-2}) \qquad (20)$$

$$\mathbf{z_n} = \mathbf{z} - \sqrt{U/K}\alpha_s\mathbf{u}(\theta) \tag{21}$$

After differentiating the natural logarithm at (20) against $\mathbf{u}(\theta)$ and the result is zero, it is obtained (22).

$$\hat{\mathbf{u}}(\theta) = \sqrt{K/U}\mathbf{z} \tag{22}$$

If the estimation in (22) is substituted for (20), which is for $\mathbf{u}(\theta)$, then (20) changes to (23).

$$P(\mathbf{z}|H_1, \alpha_n^2, \mathbf{u}(\theta)) = \frac{1}{(\pi\alpha_n^2)^{KV}} \tag{23}$$

The probability density of $\mathbf{u}(\theta)$ against H_0 is given by (24)

$$P(\mathbf{z}|H_0, \alpha_n^2) = \frac{1}{(\pi\alpha_n^2)^{KV}} exp\left(-\frac{\mathbf{z}^H\mathbf{z}}{\alpha_n^2}\right) \tag{24}$$

then the likelihood ratio test is expressed as (25)

$$ln\left(\frac{P(\mathbf{z}|H_1, \alpha_n^2, \mathbf{u}(\theta))}{P(\mathbf{z}|H_0, \alpha_n^2)}\right) = -\frac{\mathbf{z}^H\mathbf{z}}{\alpha_n^2} \tag{25}$$

so, the LRT becomes (26)-(27).

$$\|\mathbf{z}\|^2 \underset{<H_0}{\overset{>H_1}{}} \delta_P \tag{26}$$

with

$$\delta_P = \alpha_n^2 \, ln(\delta) \tag{27}$$

As in the study by [11] that the optimal detector with Neyman-Pearson criteria on the PMIMO radar is related to the noncoherent summation of MF output when θ of the signal is unknown. To obtain the detection performance on the PMIMO radar, it is assumed that θ is known so that $\mathbf{u}(\theta)$ in (18) can be substituted for MF output. Thus $\mathbf{u}(\theta)$ becomes identical and coherent integration can be formed before the detection process by multiplying the received signal vector at (7) by $[\mathbf{b}(\theta))\otimes(\mathbf{c}(\theta)\circ\mathbf{d}(\theta))]^H$ [5]. After this multiplication, the detection problem becomes (28)

$$\begin{aligned} H_0: & \quad z = n \\ H_1: & \quad z = \sqrt{U/K}U_KKV\alpha_s + n \end{aligned} \tag{28}$$

where $n \sim \mathcal{CN}(0, U_KKV\alpha_n^2)$ so that the LRT solution in (28) becomes (29).

$$\frac{P(z|H_1, \alpha_n^2, \alpha_s^2)}{P(z|H_0, \alpha_n^2)} \underset{<H_0}{\overset{>H_1}{}} \delta \tag{29}$$

Since the distributions $\mathbf{u}(\theta)$ and $\mathbf{n}$ are known, the probability density of $\mathbf{z}$ to H_1 is given by (30)

$$\begin{aligned} &P(z|H_1, \alpha_n^2, \alpha_s^2) \\ &= \frac{1}{\pi U_KKV[(U/K)U_KKV\alpha_s^2 + \alpha_n^2]} \end{aligned} \tag{30}$$

$$\times exp\left(-\frac{|z|^2}{U_KKV[(U/K)U_KKV\alpha_s^2 + \alpha_n^2]}\right)$$

and the probability density of $\mathbf{z}$ against H_0 is given by (31)

$$P(z|H_0, \alpha_n^2) = \frac{1}{\pi U_KKV\alpha_n^2} exp\left(-\frac{|z|^2}{U_KKV\alpha_n^2}\right) \tag{31}$$

then the log-likelihood ratio is expressed as (32)-(33).

$$|z|^2 \underset{<H_0}{\overset{>H_1}{}} \delta_P \tag{32}$$

with

$$\begin{aligned} &\delta_P \\ &= \frac{[UU_KKV\alpha_s^2 + K\alpha_n^2]\alpha_n^2}{U\alpha_s^2}\left(\frac{[UU_KKV\alpha_s^2 + K\alpha_n^2]\delta}{K\alpha_n^2}\right) \end{aligned} \tag{33}$$

When the target does not exist, the distribution at $|z|^2$ is exponential or the chi-square distribution with DoF is $2 \sim \chi_2^2$, according to [11], i.e. (34)

$$|z|^2 \sim exp\left(\frac{1}{U_KKV\alpha_n^2}\right) \tag{34}$$

so, the probability of false alarm for this radar i.e. (35)

$$\begin{aligned} P_{FA} &= P\left(exp\left(\frac{1}{U_KKV\alpha_n^2}\right) > \delta_P\right) \\ &= exp\left(\frac{-\delta_P}{U_KKV\alpha_n^2}\right) \end{aligned} \tag{35}$$

where the threshold as (36).

$$\delta_P = U_KKV\alpha_n^2 \, ln\left(\frac{1}{P_{FA}}\right) \tag{36}$$

If there is a target then the distribution $|z|^2$ as (37)

$$|z|^2 \sim exp\left(\frac{1}{U_KKV[(U/K)U_KKV\alpha_s^2 + \alpha_n^2]}\right) \tag{37}$$

so, the probability of detection for this radar i.e. (38)

$$P_D = exp\left(\frac{-\delta_P}{U_KKV[(U/K)U_KKV\alpha_s^2 + \alpha_n^2]}\right) \tag{38}$$

by substituting (36) on (38) it is obtained (39).

$$P_D = exp\left(\frac{K\alpha_n^2 \, ln(P_{FA})}{UU_K KV\alpha_s^2 + K\alpha_n^2}\right) \qquad (39)$$

For the probability of misdetection i.e. (40).

$$P_M = 1 - P_D \qquad (40)$$

If SNR is defined with (41),

$$SNR = \frac{\alpha_s^2}{\alpha_n^2} \qquad (41)$$

then substitute (41) on (39) so that the probability of detection becomes (42).

$$P_D = exp\left(\frac{K \, ln(P_{FA})}{UU_K KV(SNR) + K}\right) \qquad (42)$$

Equation (42), as the expressions of the detection performance of the PMIMO radar, are another main contribution of this paper. While the expressions of P_{FA} and P_D for the PA and the MIMO radars can be derived by (35) and (42) with the number of subarrays on the Tx array $1 \le K \le U$ and V elements on the Rx array because these radars are a special condition of the PMIMO radar. For radar the PA radar with $K = 1$ then $U_K = U$ so we get (43)-(44),

$$P_{FA,PA} = exp\left(\frac{-\delta_{P,PA}}{UV\alpha_n^2}\right) \qquad (43)$$

and

$$P_{D,PA} = exp\left(\frac{ln(P_{FA,PA})}{U^2V(SNR) + 1}\right) \qquad (44)$$

Meanwhile, for the MIMO radar where $K = U$ then $U_K = 1$ so that we get (45)-(46),

$$P_{FA,MIMO} = exp\left(\frac{-\delta_{P,MIMO}}{UV\alpha_n^2}\right) \qquad (45)$$

and

$$P_{D,MIMO} = exp\left(\frac{ln(P_{FA,MIMO})}{UV(SNR) + 1}\right) \qquad (46)$$

It appears that at (45)-(46) is similar to the detection performance in [10] but written differently.

The detection performance expressions state that the threshold, probability of detection, and the probability of false alarm form a one-to-one relationship. This shows that optimal detection is most determined by selecting the right threshold for the detection of the radar system target.

IV. RESULTS AND DISCUSSION

Evaluation of the performance of the parameter estimation is done by using the Capon adaptive array

method on the PMIMO radar using (13) while the LS method using (27) on [5]. Assuming the number of antenna elements in the Tx-Rx array is the same, that is $U = V = 8$, while the number of Tx subarray varies, which is $1 \le K \le U$. The spacing between antenna elements in the Tx-Rx array is half the wavelength.

Besides, a numerical evaluation of the detection performance of the PMIMO radar is given by (35) and (42) for the probability of false alarm and probability of detection, respectively. In [2] it is stated that the detection performance of radar subarrays is optimum if the threshold value is above 30 with a probability of false alarm below 10^{-4} which gives a probability of detection greater than 97% and SNR less than 10 dB.

A. Evaluation of Estimation Parameter

1) Effect of Estimator Types

To compare the performance of Capon's adaptive estimator against conventional estimators, i.e. LS, it is assumed that there are three targets located at $\Theta_A = \{-35^o, -10^o, 15^o\}$ with complex amplitude $\alpha_1 = \alpha_2 = \alpha_3 = 1$. As shown in Figure 2, the PMIMO radar ($K = 4$) using the Capon estimator has an angular resolution of the location of targets that is more accurate than the LS method. Although in terms of MCA accuracy, the LS method is relatively better than the estimator. Unlike the LS method (see Figure 2(a)), the estimator has almost no sidelobe so this condition is advantageous for overcoming strong interference and jamming (see Figure 2(b)). This fact supports the results obtained by [8] regarding the emphasis on sidelobe.

If there is a jammer around the radar then the effect on the estimated parameter performance is presented in Figure 3. It is assumed that there are three targets located at $\Theta_B = \{-55^o, -15^o, 42^o\}$ with complex amplitude $\alpha_1 = \alpha_2 = \alpha_3 = 1$ and a jammer is located at 4° with a jammer-to-noise ratio (JNR) = 20 dB. As shown in Figure 3(a), the radar is still affected by the jammer's presence as well as from other sidelobes. However, with the Capon estimator, in addition to the good resolution of the detection angle, the effect of the jammer can be suppressed and there are also no sidelobes that have the potential to cause other interference locations (see Figure 3(b)).

2) Impact of Tx Subarray Numbers

The same target conditions, i.e. Θ_A, are applied as in the previous case especially for the parameter estimation conditions by the Capon method. Figure 4 shows the effect of variations in the number of subarrays (K) on the PMIMO radar with $K = \{3, 4, 6\}$. At a glance in Figure 4(a), the number of Tx subarray does not affect the accuracy of the target angle. However, when the plot results are enlarged, it appears in Figure 4(b) that there is a difference in the detection resolution of the amplitude where for other than $K = 3$, i.e. or greater K, the performance is better, i.e. the MCA approaches the RCS value of 1. There are indications that the smaller the K, the number of target detections has the potential to be smaller, or in other words that the increase in the number of target detections is indicated by a declining MCA. The
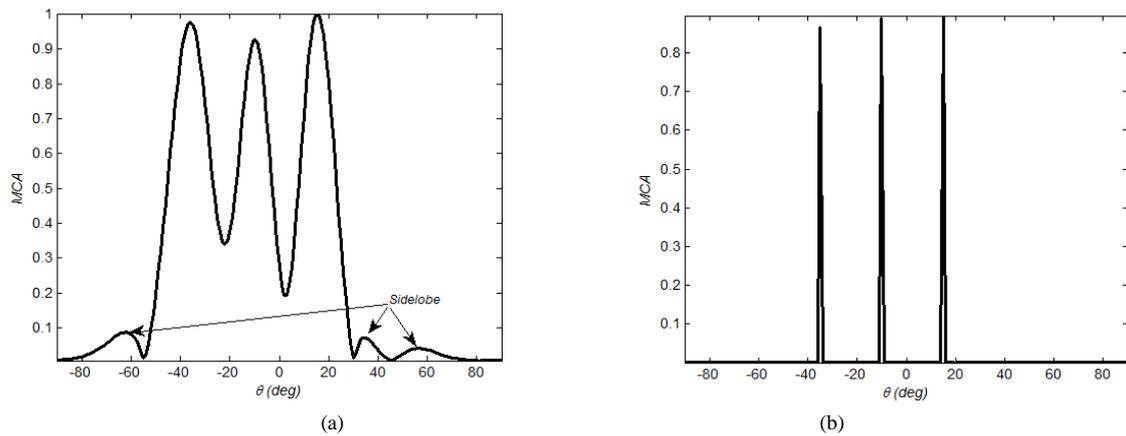
Figure 2.   The Magnitude of Complex Amplitude (MCA) or $\hat{\alpha}(\theta)$ from The PMIMO Radar ($K = 4$) with The Target Θ_A for: (a) The LS and (b) The Capon Methods.
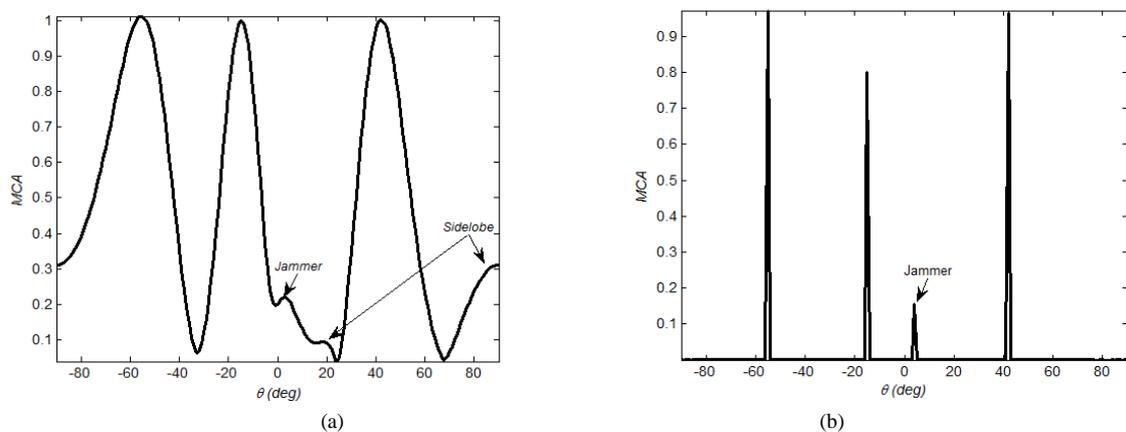
Figure 3.   The MCA from The PMIMO Radar ($K = 4$) with The Target Θ_B and Jammer for (a) The LS and (b) The Capon Methods.

study results by [2] state that at a small K, the number of targets detected is little evidenced by this study.

There is another fact that the detection angle resolution is more accurate when K is small, i.e. $K = 3$, as shown in Figure 4(b)-(d) where the locations of the three targets are detected precisely. This happens because this configuration has a narrow beamwidth of a small K of the spatial spectrum. However, the small K decreased MCA compared to the large K. Unlike the PA and the MIMO radars, this radar generally has good flexibility because it can adjust the angle and MCA resolution based on variations in the number of Tx subarrays.

3)   The Impact of Radar Types

Figure 5 presents the performance of the detection angle resolution based on the type of radar. The same assumption is used as in the previous experiment, i.e. Θ_A. It is known that the PA and MIMO radars are special conditions of the PMIMO radar where the PA radar is PMIMO ($K = 1$) while the MIMO radar is PMIMO ($K = V = 8$). The performance of the parameter estimation on the MIMO and PA radars use (16) and (17), respectively. This result supports the review of part B in this section i.e. the effect of the number of subarrays on the radar.

In Figure 5(a), it appears that the PA radar ($K = 1$) is unable to detect the targets given. Especially when seen in Figure 5(b), only the PMIMO radar ($K = 4$) and the MIMO radar ($K = 8$) can detect targets Θ_A. This is in line

with the results of studies by [2] regarding the maximum number of detectable targets where the detection ability of the PA radar is very low. It also appears that this radar target detection capability has better accuracy at angular resolution compared to the MIMO radar (Figure 5(c)). However, the detection resolution at the target magnitude, i.e. MCA, is lower than the MIMO radar. For example, comparing the accuracy of magnitude detection with the same RCS, which is $\alpha(\theta) = 1$, on this radar and the MIMO radar tabulated in Table 1.

4)   RCS Variations

Comparison of the performance estimators of the PMIMO radar ($K = 4$) against a variety of RCS is presented in Figure 6. Assuming three targets are located at $\Theta_C = \{-42°, -10°, 15°\}$ with complex amplitudes of 4, 1, and 2, respectively. For target angle resolution, the Capon estimator is clearly superior to the LS estimator (see Figure 6(a) and 6(b)). This is consistent with part A's review in this section. The accuracy of the magnitude

TABLE 1
MCA COMPARISON OF RADAR WITH VARIOUS RCS IS ONE

Target (θ)	PA ($K = 1$)	MIMO ($K = 8$)	PMIMO ($K = 4$)
-35°	NA	0.967	0.864
-10°	NA	0.982	0.888
15°	1.915e-12	0.961	0.896

Note: NA denotes non-available
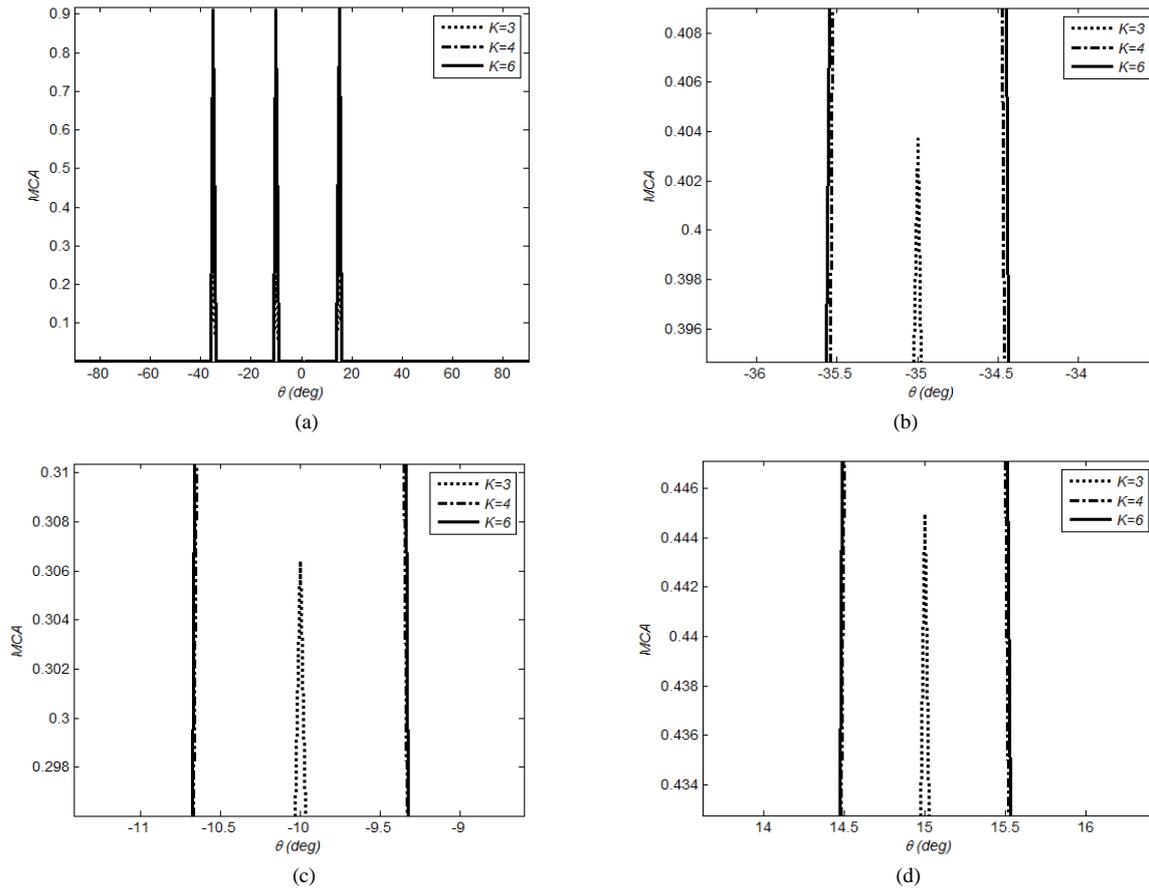
Figure 4. The MCA from Capon Method of The PMIMO Radar with Variation of K and The Target Θ_A for (a) Real and (b) Magnified Condition at -35°, (c) Magnified Condition at -10°, and (d) Magnified Condition at 15°.

detection for the three complex amplitudes in these targets means that the LS estimator performance is significantly better than the Capon estimator because each has a root mean square error (RMSE) detection magnitude of 0.033 and 0.062, respectively.

B. Evaluation of Detection Performance

1) Effect of SNR

It has been mentioned by [11] that the detection performance of the PA radar is better than the MIMO radar for low SNR. Evaluation of detection performance to calculate the probability of false alarm and the probability of detection on the PA radar, i.e. (43) and (44) and the MIMO radar, i.e. (45) and (46). Using the same assumption from the previous performance evaluation with an SNR range of -15 dB - +15 dB, the evaluation results in Figure 7(a) support the results given by [11] that the detection performance of the PA radar is better than the MIMO radar. For SNR above 10 dB, the detection performance is above 95%. It also appears that the detection performance of this radar has the ability to vary the number of subarrays, namely K, which can

adjust the P_D value according to the target condition, which is close to the PA radar performance. Figure 7(b) presents the detection performance, i.e. the probability of misdetection (P_M), for the PMIMO radar as a P_{FA} function for SNR variations. For SNR greater than 5 dB, the P_M performance is smaller than 10^{-4} which indicates P_D above 99%. This is as reported by [2].

2) Impact of the Number of Subarrays on Tx

Since the threshold, P_{FA}, and SNR are fixed, the detection performance of the radar, i.e. P_D, is determined by the number of subarrays (K) in the Tx array. This shows the advantages of this radar compared to other radars, i.e. its high flexibility in adjusting detection performance to target conditions. Figure 8 presents the detection performance of this radar (P_D) for the variation of K. It appears that the detection performance (P_D) increases with increasing K. If given K, i.e. {1, 2, 4, 6, 8, 10} then P_D at $K = 1$ has better performance than P_D for other variations of K. Thus, the detection capability of the PMIMO radar can be carried out by adjusting the number of subarrays in Tx (K) based on the detected multi-target conditions which other types of radar are unable to do.

(a)

(b)

(c)
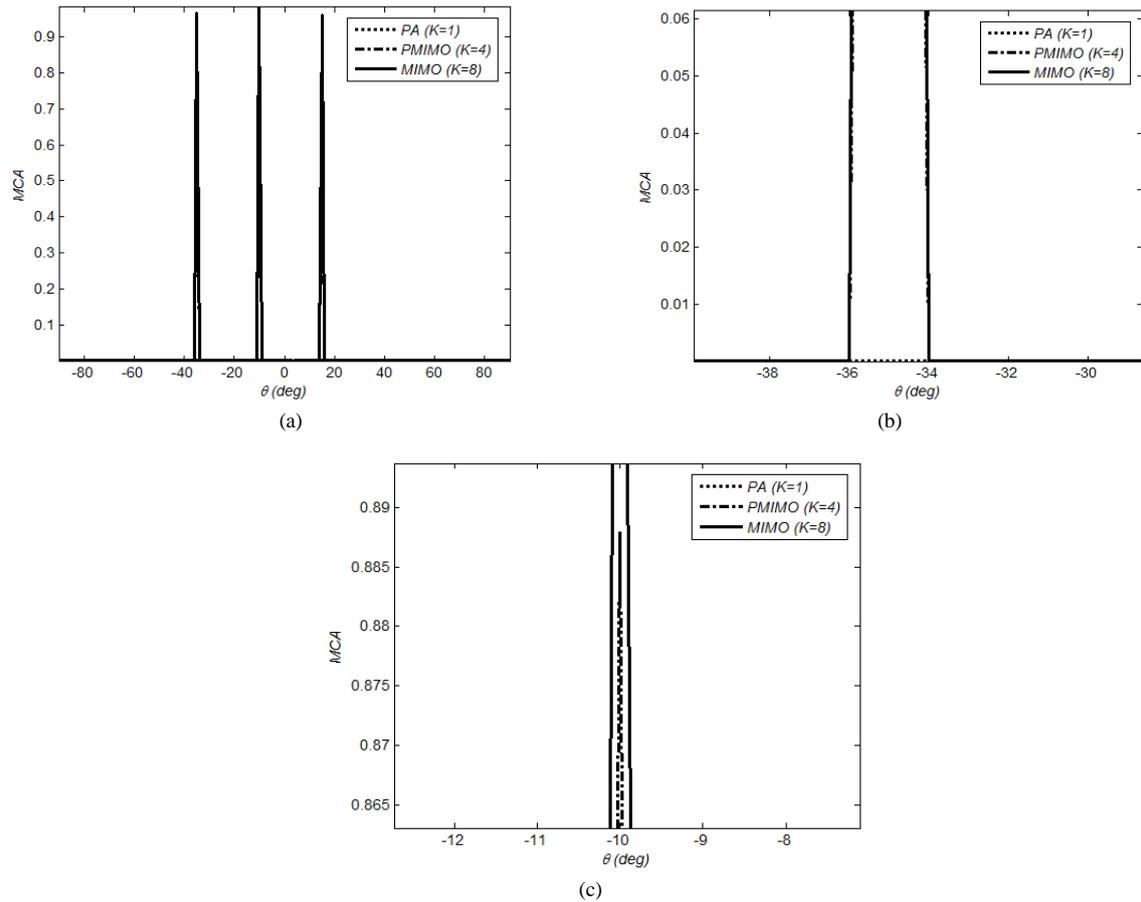
Figure 5. The MCA from Capon Method of Radar Types with The Target Θ_A for (a) Real, (b) Magnified Condition at -35º, and (c) Magnified Condition at -10º.

(a)

(b)
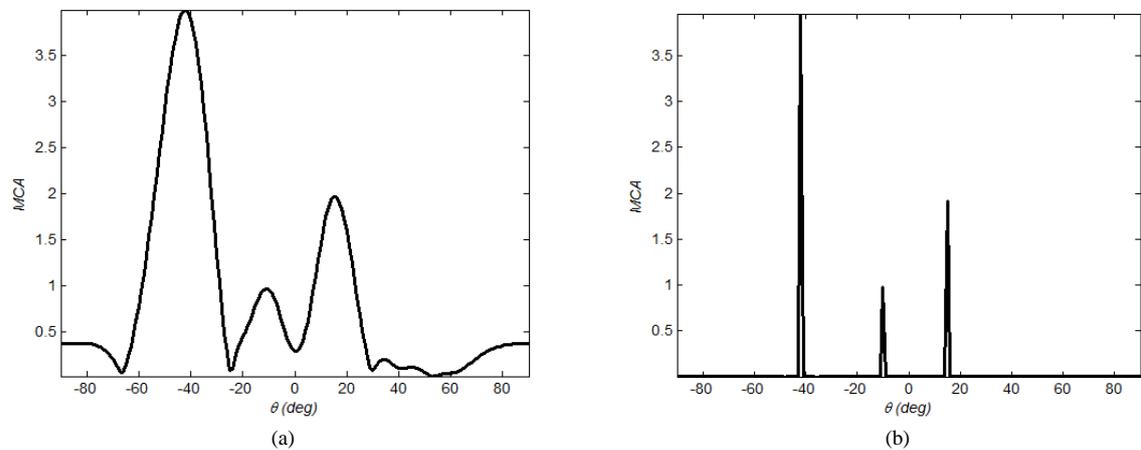
Figure 6. The MCA of PMIMO Radar ($K = 4$) with Variation of RCS and The Target Θ_C for (a) The LS and (b) Capon Methods.

CONCLUSION

This paper has formulated the parameter estimation of target detection by the Capon method and detection performance with the LRT approach. The performance of these estimators has been compared to conventional estimators such as LS estimators by considering factors such as the number of Tx subarrays, radar types, and RCS variations of the target detected. Detection angle resolution using this method is relatively better than other radar performance. However, the amplitude detection performance of this target estimator has an RMSE below the LS estimator, i.e. 0.033 and 0.062, respectively. Therefore, the magnitude detection performance of this estimator needs to be improved to be more effective with other estimators. In general, the detection angle resolution obtained by this estimator has almost no sidelobe, so this condition is advantageous for the design of the radar system, especially when overcoming strong interference and jamming. Meanwhile, in the detection performance to achieve the performance target that fulfills a certain threshold tolerance, which is above 30,
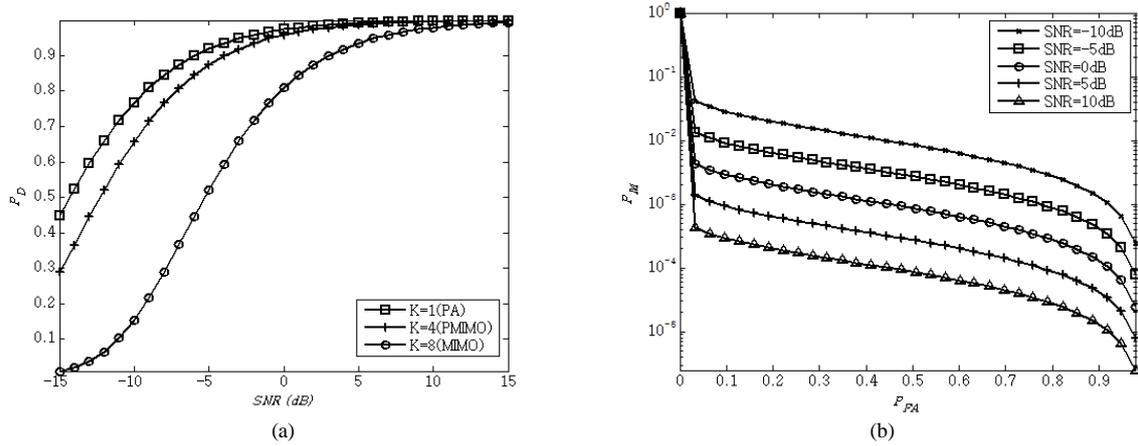
Figure 7. (a) P_D vs. SNR of radars for $P_{FA} = 10^{-4}$ and (b) P_M vs. P_{FA} for various SNRs.
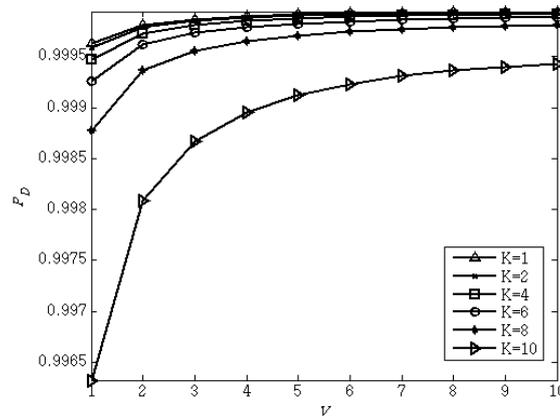
Figure 8. P_D vs. the total number of elements on Rx (V) of the PMIMO radar for $\delta = 30$, $\alpha_p^2 = 1$, $\alpha_n^2 = 0.1$, $P_{FA} = 10^{-4}$ and the variation of K.

the resulting P_{FA} value is above 10^{-4} and P_D is around 99%. The advantages and flexibility of this radar to detect targets are determined by the variation in the number of subarrays in Tx, i.e. K, where a small number of K will give a high P_D value.

REFERENCES

[1] I. Bilik, O. Longman, S. Villeval, and J. Tabrikian, "The rise of radar for autonomous vehicles: signal processing solutions and future research directions," *IEEE Signal Process. Mag.*, vol. 36, no. 5, pp. 20–31, Sep. 2019.

[2] S. Tahcfulloh and G. Hendrantoro, "FPMIMO: a general MIMO structure with overlapping subarrays for various radar applications," *IEEE Access*, vol. 8, pp. 11248–11267, Jan. 2020.

[3] H. Pratiwi, M. R. Hidayat, A. A. Pramudita, and F. Y. Suratman, "Improved FMCW radar system for multi-target detection of human respiration vital sign," *Jurnal Elektronika Telekomunikasi*, vol. 19, no. 2, pp. 38–44, Dec. 2019.

[4] A. Hassanien and S. A. Vorobyov, "Phased-MIMO radar: a tradeoff between phased-array and MIMO radars," *IEEE Trans. Signal Process.*, vol. 58, no. 6, pp. 3137–3151, Jun. 2010.

[5] M. Hardiwansyah, S. Tahcfulloh, and G. Hendrantoro, "Parameter identifiability of phased-MIMO radar," in *Proc. Int.*

Conf. Artificial Intell. Inform. Technol., Yogyakarta, Indonesia, Mar. 2019, pp. 192–195.

[6] L. Xu, J. Li, and P. Stoica, "Target detection and parameter estimation for MIMO radar systems", *IEEE Trans. Aerosp. Electron. Syst.*, vol. 44, no. 3, pp. 927–939, Jul. 2008.

[7] M. R. Widyantara, Sugihartono, F. Y. Suratman, S. Widodo, and P. Daud, "Analysis of non linear frequency modulation (NLFM) waveforms for pulse compression radar," *Jurnal Elektronika Telekomunikasi*, vol. 18, no. 1, pp. 27–34, Aug. 2018.

[8] C. Gao, H. Zhou, R. Wu, X. Xu, F. Shen, and Z. Guo, "Parameter estimation and multi-pulse target detection of MIMO radar," in *Proc. 2016 IEEE Region 10 Conf.*, Singapore, Nov. 2016, pp. 909–914.

[9] J. Li and P. Stoica, *MIMO Radar Signal Processing*. Hoboken, NJ: John Wiley & Sons, Inc., 2009.

[10] A. K. M. T. Rahman, S. M. M. H. Mahmud, T. K. Biswas, and S. Naznin, "Target detection performance of coherent MIMO radar using space time adaptive processing," in *Proc. 2014 Int. Conf. Informatics Electron. Vision*, Dhaka, Bangladesh, 2014, pp. 1–5.

[11] E. Fishler, A. Haimovich, R. S. Blum, L. J. Cimini, D. Chizhik, and R. A. Valenzuela, "Spatial diversity in radars-models and detection performance", *IEEE Trans. Signal Process.*, vol. 54, no. 3, pp. 823–838, Mar. 2006.